Left-right asymmetry in semi-inclusive deep inelastic scattering process

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Abstract

We analyze the left-right asymmetry of pion production in semi-inclusive deep inelastic scattering (SIDIS) process of unpolarized charged lepton on transversely polarized nucleon target. Unlike available treatments, in which some specific weighting functions are multiplied to separate theoretically motivated quantities, we do not introduce any weighting function following the analyzing method by the E704 experiment. The advantage is that this basic observable is free of any theoretical bias, although we can perform the calculation under the current theoretical framework. We present numerical calculations at both HERMES kinematics for the proton target and JLab kinematics for the neutron target. We find that with the current theoretical understanding, Sivers effect plays a key role in our analysis.

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1 Introduction

Single spin asymmetries (SSAs) on a transversely polarized target provide us with rich information on the spin structure of the nucleon, especially on the transverse spin. However, there has been a prejudice that all transverse spin effect should be suppressed at high energies in the past. It was not until in the 1990s, when the E704 Collaboration reported their observation of a left-right asymmetry in $p^\uparrow p \to \pi X$ process [1], that people began to show enthusiasms on transverse spin effects. In order to account for the asymmetry, Sivers [2] suggested a possible mechanism, which is now called "Sivers effect", originating from the asymmetry of the distribution function. But this idea was criticized by Collins [3] on the ground of violating the time reversal invariance of QCD. In Ref. [3,4], another possible explanation, that asymmetry arises from a fragmentation which is now known as "Collins effect", was proposed. However, in Ref. [5], it was argued that Sivers asymmetry might be allowed, and a good description of E704 experiment was obtained by a parametrization. In Ref. [6], another good description of E704 data was obtained, but based on the Collins effect this time with a surprising large contribution from unfavored fragmentation. Remember that in Ref. [7], the calculation is not so good to reproduce the data based on Collins effect with the naive assumption of favored fragmentation dominance. Later, the suppression of Collins mechanism is also reproduced in Ref. [8] by incorporating the intrinsic partonic motion together with correct azimuthal angular dependence. Now we have learnt [9,10] that there are three possible mechanisms contributing to the $p^\uparrow p \to \pi X$ process: the Sivers effect, the Collins effect and the Boer-Mulders effect [11]. In Ref. [10], it was pointed that the Sivers effect is important and other effects might be suppressed. We should also aware that there is an alternative attempt to explain the left-right asymmetry by the valence quark orbital angular moment effect [12], in distinct from the introduction of new distribution and fragmentation functions.

Due to the complexity of the hadron-hadron process, we might as well turn our point to a simpler process, the semi-inclusive deep inelastic scattering (SIDIS) process, which has attracted many interests in recent years. Meanwhile, many progresses have been made by experiments, e.g., non-vanishing SSAs have been observed by HERMES [13] and COMPASS [14] collaborations. On the theoretical side, we have known that both Sivers and Collins effects may contribute to the asymmetry. By multiplying different weighting functions, the two effects can be separated, which is now the conventional way of analyzing the data. Nevertheless, the selection of the weighting functions strongly shows our bias on the current theory. So in this paper, we will analyze the basic quantity of left-right asymmetry in SIDIS process, following the analyzing method by the E704 experiment, in which no weighting functions were multiplied, to see whether a non-zero asymmetry can be obtained. With the current theoretical

knowledge, we find that the Sivers effect plays a key role is our numerical calculation and indeed produces a sizable left-right asymmetry in $\pi^{\pm}$ production process. Therefore we suggest to measure the left-right asymmetries in SIDIS process, for the purpose to provide a basic observable for theoretical studies.

2　Definition of the asymmetry

In the E704 experiment [1], the left-right asymmetry is defined as:

$$A = -\frac{1}{P_B \langle \cos \phi \rangle} \frac{N_\uparrow(\phi) - N_\downarrow(\phi)}{N_\uparrow(\phi) + N_\downarrow(\phi)}. \tag{1}$$

P_B is the beam polarization and ϕ is the azimuthal angle between the beam polarization direction and the normal to the $\pi^{\pm}$ production plane. $N_{\uparrow(\downarrow)}$ is the number of pions produced for beam spin tagged as positive (negative) normalized to the beam flux.

Following the similar method, we define our asymmetry for the SIDIS process as:

$$A(\psi_s) = \frac{1}{S_T} \frac{N(\psi_s) - N(\psi_s + \pi)}{N(\psi_s) + N(\psi_s + \pi)} = \frac{1}{S_T} \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}. \tag{2}$$

S_T is the transverse polarization of the target; ψ_s is the azimuthal angle between the transverse spin vector plane (defined by spin vector and the incident beam) and a definite plane. The definite plane can be chosen arbitrarily, e.g., we can choose the horizontal plane in the laboratory frame for convenience. If integrating the cross sections in the numerator and denominator separately, we can investigate the asymmetry depending on various kinematical variables.

Here we emphasize our difference with the conventional treatment. When we perform the integration, no weighting functions are multiplied, so we cannot integrate the azimuthal angles for the produced hadrons from 0 to 2π, which must lead to a vanishing result. Instead, we will limit the azimuthal angles in a certain range, e.g., $-\frac{\pi}{4}$ to $\frac{\pi}{4}$ (or $\frac{3\pi}{4}$ to $\frac{5\pi}{4}$), i.e., only the hadrons produced in a range to the left (right) of the spin plane will be selected, which is the way E704 experiment dealt with the data. This detected region changes from left (right) to right (left) as the target spin changes from up to down, thus a left-right asymmetry is obtained. However, we have two choices to define the spin plane. In E704 experiment, this plane was defined by the incident beam and the spin vector, but in our paper, this plane is defined by the virtual photon and the spin vector. We believe this is reasonable and acceptable, for the DIS

process can be considered as a virtual Compton scattering process. So for the convenience of theoretical description, the direction of the virtual photon is chosen as the z-axis, which is denoted as the $\gamma^* p$ frame. Correspondingly, ℓp frame denotes the frame where the lepton beam is defined as the z-axis. We can transform from one coordinate system to another via a rotation by the angle θ between the exchanged photon and the incident beam. We have [15]:

$$\sin\theta = \gamma\sqrt{\frac{1 - y - \frac{1}{4}y^2\gamma^2}{1 + \gamma^2}}, \qquad \gamma = 2xM_p/Q. \tag{3}$$

If x is small, this angle is also small, which means that the incident beam and the virtue photon almost lay in the same direction. We make a rough estimation for HERMES experiment [13]: $\langle x \rangle = 0.09$, $\langle y \rangle = 0.54$, $\langle z \rangle = 0.36$, $\langle Q^2 \rangle = 2.41\text{GeV}^2$, thus we have $\langle \sin(\theta) \rangle \approx 0.073$, which is indeed very small. But we should be careful that as x increases, this angle might not be ignored.

3   Expressions of the cross sections

Due to the existence of the angle θ, the component of a vector can be different in different frames. For a transversely polarized target, the polarization direction is perpendicular to the incident beam, so the spin vector does not have the parallel component in the ℓp frame. But in the $\gamma^* p$ frame, a parallel component of the spin vector is projected along the z-axis, which means that we have longitudinal effect here although the target is transversely polarized. By taking into account this factor, the cross section up to leading twist is given as follows [15,16]:

$$\frac{d\sigma}{dxdyd\phi_S^\ell dzd\phi_h^\ell dP_{h\perp}^2} = \frac{\alpha^2}{2sx(1-\epsilon)}\frac{\cos\theta}{1 - \sin^2\theta\sin^2\phi_s^\ell} \times \left\{ \mathcal{F}[f_1 D_1] \right.$$
$$- \frac{S_T\cos\theta}{\sqrt{1 - \sin^2\theta\sin^2\phi_s^\ell}}\sin(\phi_h^\ell - \phi_s^\ell)\mathcal{F}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{M_p}f_{1T}^\perp D_1\right]$$
$$\left. - \frac{S_T\cos\theta}{\sqrt{1 - \sin^2\theta\sin^2\phi_s^\ell}}\sin(\phi_h^\ell + \phi_s^\ell)\mathcal{F}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M_h}h_1 H_1^\perp\right] \right\}$$
$$\equiv d\sigma_{UU} + d\sigma_{Siv} + d\sigma_{Col}, \tag{4}$$

where

4

$$\epsilon = \frac{1 - y - \frac{1}{4}y^2\gamma^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}y^2\gamma^2}, \qquad \hat{\boldsymbol{h}} \equiv \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|. \tag{5}$$

The angles ϕ_h^ℓ and ϕ_s^ℓ are defined as: $\phi_h^\ell = \phi_h - \phi^\ell$, $\phi_s^\ell = \phi_s - \phi^\ell$, where ϕ^ℓ denotes the orientation angle of the lepton plane. Notice here that all the angles appearing in the cross section are defined in the $\gamma^* p$ frame. In the above formula, we use a compact notation:

$$\mathcal{F}[\omega f D] = \sum_a e_a^2 \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{k}_\perp \delta^2(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp - \boldsymbol{P}_{h\perp}/z)\omega(\boldsymbol{p}_\perp, \boldsymbol{k}_\perp)f^a(x, p_\perp^2)D^a(z, z^2 k_\perp^2), \tag{6}$$

where $\omega(\boldsymbol{p}_\perp, \boldsymbol{k}_\perp)$ is an arbitrary function. The factors depending on θ before relevant terms are due to the transformation from $\gamma^* p$ to ℓp frames.

First, we may change the integration variables from $d\phi_S^\ell d\phi_h^\ell$ to $d\phi^\ell d\phi_h$, and we can perform the integration over ϕ^ℓ. We notice that

$$\sin(\phi_h^\ell - \phi_s^\ell) = \sin(\phi_h - \phi_s),$$
$$\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp = p_\perp \cos(\phi_h - \phi_{p_\perp}),$$
$$\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp = k_\perp \cos(\phi_h - \phi_{k_\perp}), \tag{7}$$

all of which are independent of ϕ^ℓ, but

$$\sin(\phi_h^\ell + \phi_s^\ell) = \sin(\phi_h + \phi_s - 2\phi^\ell),$$
$$\sin \phi_S^\ell = \sin(\phi_S - \phi^\ell), \tag{8}$$

both of which depend on ϕ^ℓ. If we ignore the difference between the $\gamma^* p$ and ℓp frame, we have $\sin\theta = 0$, $\cos\theta = 1$. After integration over ϕ^ℓ, only the Sivers effect survives, and all the other terms including the Collins term vanish. With a more strict management, we will not ignore θ, but expand the factors in $\sin^2\theta$, then we have:

$$\frac{1}{2\pi} \int_0^{2\pi} d\phi^\ell \frac{1}{1 - \sin^2\theta \sin^2\phi_S^\ell} = 1 + \frac{1}{2}\sin^2\theta + o(\sin^4\theta),$$
$$\frac{1}{2\pi} \int_0^{2\pi} d\phi^\ell \frac{\sin(\phi_h^\ell - \phi_S^\ell)}{(1 - \sin^2\theta \sin^2\phi_S^\ell)^{3/2}} = \sin(\phi_h - \phi_S)(1 + \frac{3}{4}\sin^2\theta + o(\sin^4\theta)),$$
$$\frac{1}{2\pi} \int_0^{2\pi} d\phi^\ell \frac{\sin(\phi_h^\ell + \phi_S^\ell)}{(1 - \sin^2\theta \sin^2\phi_S^\ell)^{3/2}} = -\sin(\phi_h - \phi_S)(\frac{3}{8}\sin^2\theta + o(\sin^4\theta)) \tag{9}$$

We find that the Sivers effect is $o(1)$, but the Collins effect is $o(\sin^2\theta)$, which means that it is suppressed by $1/Q^2$. Generally, only the terms independent

of ϕ^ℓ are $o(1)$, and all the other effects are suppressed by $1/Q^2$, so Sivers effect is dominant in our analysis, which is coincident with the analysis in Ref. [10].

In our calculation, we select the produced hadrons within the range $\frac{3}{4}\pi \leqslant \phi_h \leqslant \frac{5}{4}\pi$, the right side of the spin plane. Also we can choose the left side, and it is clearly the same as we can see from the expression of the cross section. Finally, we write the asymmetry for our numerical calculation:

$$A_{UT}(x,y,z) = \frac{\int d\phi_S^\ell dP_{h\perp}^2 d\phi_h^\ell \, (d\sigma_{Siv} + d\sigma_{Col})}{\int d\phi_S^\ell dP_{h\perp}^2 d\phi_h^\ell \, d\sigma_{UU}}. \tag{10}$$

4   Numerical calculations

To perform the calculation, we first need an input of Sivers functions. However, there could be non-universality of transverse momentum dependent distributions in different processes [17], e.g., the Sivers asymmetry may enter in hadron process with specific factors rather than simply a sign change from SIDIS process. Therefore we should be cautious to apply the parametrization extracted from one process to other kind of processes [18]. Fortunately, what we will calculate is for the SIDIS process, and the parametrization of Sivers functions is also from SIDIS data in Ref. [19,20], in which the Sivers function is parameterized in the form:

$$f_{1T}^{\perp q}(x,p_\perp^2) = -\frac{M_p}{p_\perp}\mathcal{N}_q(x)f_q(x)g(p_\perp^2)h(p_\perp^2), \tag{11}$$

$$\mathcal{N}_q(x) = N_q x_q^a (1-x)_q^b \frac{(a_q+b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}}, \tag{12}$$

$$g(p_\perp^2) = \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi\langle p_\perp^2 \rangle}, \quad h(p_\perp^2) = \sqrt{2e}\frac{p_\perp}{M'}e^{-p_\perp^2/\langle M'^2 \rangle}. \tag{13}$$

$f_1(x)$ is the unpolarized parton distribution functions, and we adopt the CTEQ6L parametrization [21] as an input. We plot $f_{1T}^{\perp(1)q}(x)$, the one-moment of the Sivers function in Fig. 1. This parametrization seems to indicate that $|f_{1T}^{\perp(1)d}(x)| > |f_{1T}^{\perp(1)u}(x)|$, so we expect a larger asymmetry in a neutron target than that in a proton target.

The fragmentation functions are [22]:

$$\begin{aligned} D_{\mathrm{fav}}(z) &= 0.689 z^{-1.039}(1-z)^{1.241}, \\ D_{\mathrm{unf}}(z) &= 0.217 z^{-1.805}(1-z)^{2.037}. \end{aligned} \tag{14}$$
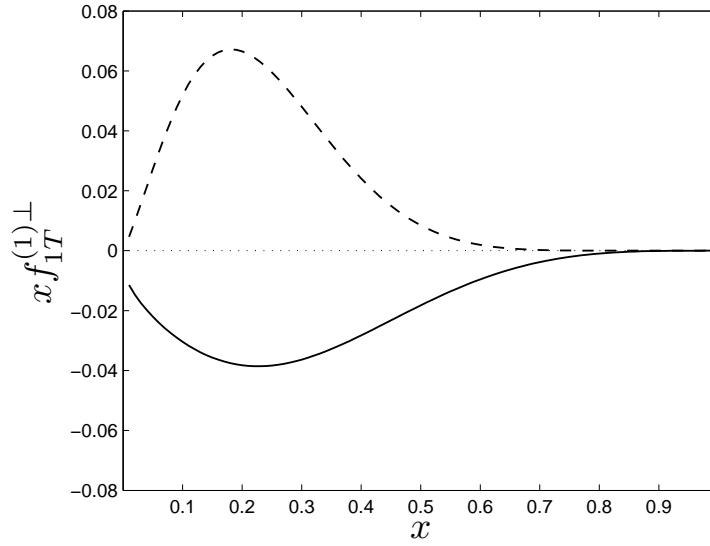
Fig. 1. $xf_{1T}^{\perp(1)q}(x)$ for u and d quarks in a proton. The solid and dashed curves correspond to u and d quarks respectively.

Table 1
kinematics

HERMES	JLab
$s = 51.7\text{GeV}^2$	$s = 23.4\text{GeV}^2$
$Q^2 > 1\text{GeV}^2$	$Q^2 > 1\text{GeV}^2$
$W^2 > 10\text{GeV}^2$	$W^2 > 4\text{GeV}^2$
$0.023 < x < 0.4$	$0.05 < x < 0.55$
$0.1 < y < 0.85$	$0.34 < y < 0.9$
$0.2 < z < 0.7$	$0.3 < z < 0.7$

In our calculation, we will consider the Collins effect, but as we argued before that Collins effect is suppressed in our analysis, so we will not care about the details on transversity and the Collins functions, which are not known clearly yet. We will use the SU(6) quark-diquark model [23] by including the Melosh-Wigner rotation effect [24] to describe transversity and adopt the parametrization of Collins functions given by Ref. [25].

The kinematical cuts used in the calculation are shown in Table 1.

For the HERMES experiment, a proton target is assumed, while for the Jefferson Lab (JLab) experiment, a neutron target is assumed. We will investigate the x and z dependence [1] of the asymmetries for both π^+, π^- and π^0 produc-

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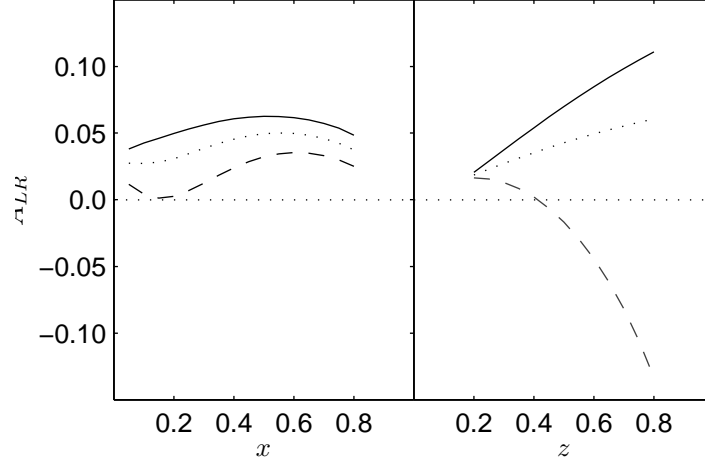[1] The E704 experiment only showed the dependence on x_F, i.e. approximate z here.

Fig. 2. Asymmetries for π production at HERMES kinematics. The solid, dashed and dotted curves correspond to the results for the π^+, π^- and π^0 production respectively. A proton target is assumed here.
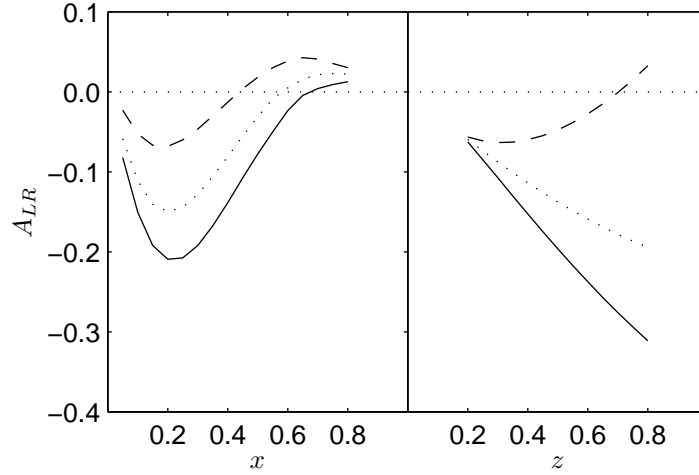

Fig. 3. The same as Fig. 2, but a neutron target is assumed here.

tions.

Fig. 2 shows the results for π production on a transversely polarized proton target at HERMES kinematics, and Fig. 3 shows the same results, but on a transversely polarized neutron target at JLab kinematics. From these figures, we clearly show non-vanishing asymmetries depending on x and z. Firstly, we notice that the asymmetries for π^+, π^- and π^0 productions are different, especially for the z-dependence of the asymmetry, which is quite similar to that in the E704 experiment. This can be accounted for by different frag-

mentation functions for different meson production. Secondly, the result for a neutron target behaves completely different, almost opposite to that for a proton target. If we notice that the Sivers functions for u and d quarks are of different signs, this can be deduced directly from the isospin symmetry between the proton and the neutron. The parametrization we used indicates that the Sivers distribution for d quarks is a little larger than that for u quarks, thus a larger asymmetry is obtained in a neutron target as the figures have shown. However, we should be careful about it, and the correctness of the parametrization needs a further check.

5   Conclusion

Single spin asymmetry (SSA) is a powerful instrument to explore the internal structure of the nucleon. A lot of theoretical works have tried to obtain the asymmetries, and under the guidance, recent experiments reported their discovery of the asymmetries. According to the conventional treatment, various weighting functions should be multiplied to project out the corresponding asymmetries. However, the choice of a weighting function strongly shows a bias on a certain theory, e.g., the current parton model based on operator product expansion (OPE) and factorization. We do not consider it a natural way dealing with the data, and it may not work if the theory changes. In fact, there exist other theories such as the recombination model [26,27] which can explain the spin structure of the nucleon and the SSA phenomena. We expect a "universal" observable independent of any theory, and fortunately, E704 experiment provided us an example.

In this paper, we analyzed the SIDIS process, following the method by the E704 experiment. Our result clearly showed a left-right asymmetry, with no weighting functions multiplied. Under the current theoretical framework, we found that Sivers effect plays the key role in our analysis, which might be helpful to understand the E704 experiment. We should emphasize that although our calculation depends on the current theory, the basic observable of left-right asymmetry is free of bias on any theories or models. We give the predictions at both HERMES and JLab kinematics, and we suggest that relevant experimental collaborations deal with their data in this way to provide more information for theoretical studies.

Acknowledgement

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